Domination in tournaments

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Let T be a tournament. A *dominating set* of T is a set of vertices S such that $N^+[S] = V(T)$. The size of a smallest dominating set is denoted by $\gamma(T)$. T may contain an induced subgraph with domination number larger than $\gamma(T)$. We therefore define the hereditary domination number: $\gamma_H(T) := \max_{X \subseteq V} \gamma(T[X])$.

We propose the following property saying that if the out-neighborhood of every vertex of a tournament has bounded hereditary domination number, then the tournament has bounded domination number.

Conjecture 1. *Let $T = (V, E)$ be a tournament. If $\gamma_H(N^+(v)) \leq t$ for all $v \in V$, then $\gamma(T) \leq f(t)$.*

This conjecture is a weakening of the following conjecture proposed by Harutyunyan, Le, Thomassé, and Wu, saying that if a tournament has very large domination number, then it contains a bounded size tournament with large domination number. In other words, Conjecture 2 implies Conjecture 1.

Conjecture 2. *Let $T = (V, E)$ be a tournament. If $\gamma(T) > k$, then there is a subset $S \subset V$ such that $|S| \leq g(k)$ and $\gamma(T[S]) \geq h(k)$.*

References

[1] A. Harutyunyan, T.-N. Le, S. Thomassé, and H. Wu, Coloring tournaments: from local to global, *arXiv:1702.01607*.